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Translation invariance, commutation relations and ultraviolet/infrared mixing

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Abstract: We show that the Ultraviolet/Infrared mixing of noncommutative field theories with the Grönewold-Moyal product, whereby some (but not all) ultraviolet divergences become infrared, is a generic feature of translationally invariant associative products. We find, with an explicit calculation that the phase appearing in the nonplanar diagrams is the one given by the commutator of the coordinates, the semiclassical Poisson structure of the non commutative spacetime. We do this with an explicit calculation for represented generic products.

Keywords: Non-Commutative Geometry, Space-Time Symmetries

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Dedicated to the memory of Raffaele Punzi

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Contents

1 Introduction

One of the original motivations [1, 2] to consider a noncommutative structure of space or spacetime was the hope that the presence of a dimensionful parameter, and a modification of the short distance properties, could resolve the problem of the infinities of quantum field theory. The analogy in this case is the presence of $\hbar$ and the noncommutativity of quantum phase space solves the so called ultraviolet catastrophe of the black body radiation. In the case of a field theory described by the Grönewold-Moyal product this hope is not fulfilled. In this case instead of the elimination (at least partial) of the ultraviloet infinities, we enconter the phenomenon of ultraviolet/infrared mixing [9], one of the novel features of a field theory over a noncommutative space (noncommutative field theory).

Technically this means that some ultraviolet divergences of the ordinary theory disappear, at the price of the appearance of infrared divergences in the same diagrams. In particular one finds that this happens at one loop for nonplanar diagrams. Therefore while the ultraviolet, short distance, properties of the theory are changed in the sense of a mitigation of the infinities, the price paid is the appearance of new kind of infinity. We will describe in detail this phenomenon for the one loop corrections to the propagator, but there is also a rough heuristic explanation of this phenomenon. The noncommutative $\star$ product used in the Grönewold-Moyal product reproduces the commutation relation of quantum mechanics: $[x^i, x^j] = \mathrm{i}\theta^{ij}$. Coordinates do not commute and therefore a generalization of Heisenberg's uncertainty principle is at work, a small uncertainty in the x^i direction implies a great uncertainty in the $\theta^{ij}x_j$ direction. Therefore a short distance in one direction and the long distance in the other are coupled. This reasoning can be made more precise [9] (but still heuristic) considering the dispersion of the product of gaussian functions. The phenomenon persists also in the nonrelativistic case [10].

The aim of this paper is to discuss the ultraviolet structure of noncommutative theories with more general products than Grönewold-Moyal. Our analysis will be centered on the one loop correction to the propagator, which is the source of all mixing. We will discuss only the bosonic ϕ^4 theory, but the results are more general than that and will apply to other scalar and gauge theories as well since, as we will see, the behaviour which we find is quite generic.

We will be discussing the euclidean version of the theory, or equivalently the case of only spatial commutativity. In the Minkowskian case the nonlocality of the theory has been claimed to lead to loss of unitarity [3] in the noncommutative theory which is obtained as an effective theory of strings [4]. Nevertheless for theories for which noncommutativity is fundamental there are issues of time ordering [5–8] which show that an appropriate treatment can lead to an unitary theory. For purely time-space noncommutativity the mixing may not present as such [17].

The ultraviolet/infrared mixing is in general connected with the nonlocality of the product and has been generalised in various directions. Gayral [16] has shown that it persists in the presence of isospectral deformations. The noncommutativity for the compact case is basically given by a noncommutative torus, which in this context is a compact version of the Grönewold-Moyal product. Some form of mixing also survives for the κ-Minkowski case [15].

The paper is organized as follows. In section 2 we discuss the Ultraviolet/Infrared Mixing for the Grönewold-Moyal Product. We then discuss the general form of translationally invariant products. In section 4 we show the form of the mixing for a general product. This section contains the main result of the paper, that is that the mixing persists unchanged for a generic translation invariant product. We end the paper with some conclusions.

2 Ultraviolet/infrared mixing for the Grönewold-Moyal product

In this section we review the presence of ultraviolet/infrared mixing for a scalar theory. We consider a field theory on a noncommutative space described by the action:

$$S = \int \mathrm{d}x^d \frac{1}{2}\left(\partial_i\varphi \star \partial_i\varphi - m^2\varphi\star\varphi\right) + \frac{g}{4!}\varphi\star\varphi\star\varphi\star\varphi \tag{2.1}$$

where $\star$ usually denotes the Grönewold-Moyal product between functions which can be defined in several different ways. These definitions are equivalent up to the fact that the domain of definition can be different. The product depends on an antisymmetric matrix θ^{ij} and we write two standard expressions of it. The most common expression is expressed as a series of differential operators:

$$(f \star_M g)(x) = \mathrm{e}^{\frac{\mathrm{i}}{2}\theta^{ij}\partial_{y_i}\partial_{z_j}} f(y)g(z)\Big|_{x=y=z} \tag{2.2}$$

This series is an asymptotic expansion [11] of (equivalent) integral expressions, some of which can be found in the appendix of [13]. For the purposes of this paper the useful form of the product is the following:

$$(f \star_M g)(x) = \frac{1}{(2\pi)^{\frac{d}{2}}} \int \mathrm{d}^d p\, \mathrm{d}^d q \tilde{f}(q)\tilde{g}(p-q)\mathrm{e}^{\mathrm{i}p\cdot x}\mathrm{e}^{\frac{\mathrm{i}}{2}p_i\theta^{ij}q_j} \tag{2.3}$$

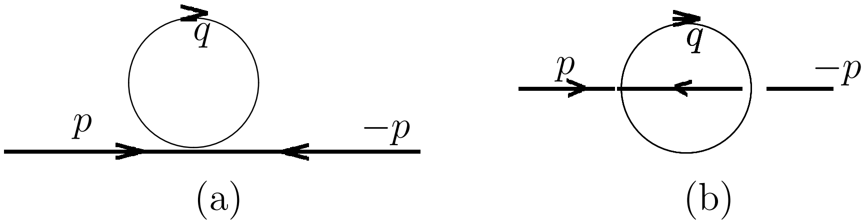

Figure 1. The planar (a) and nonplanar (b) one loop correction to the propagator

where $\tilde{f}$ and $\tilde{g}$ are the usual Fourier transforms of f and g respectively. In both cases it results

$$[x^i, x^j]_{\star_M} = \mathrm{i}\theta^{ij} \tag{2.4}$$

and the product becomes the ordinary, commutative product for $\theta = 0$. Note that for this product

$$\int \mathrm{d}x^d f \star_M g = \int \mathrm{d}x^d f g \tag{2.5}$$

which means that the free (quadratic) theory is the same in the commutative and noncommutative cases.

The theory described by the action (2.1) has a propagator which is the same as in the commutative case and a vertex [14] which is easily calculated from (2.3) to be, for four incoming propagators of momenta k_a,

$$V_{\text{Moyal}} = V_0 \mathrm{e}^{-\frac{\mathrm{i}}{2}\sum_{a\leq b}\theta^{ij}k_{a\,i}k_{b\,j}} \tag{2.6}$$

where

$$V_0 = -\mathrm{i}\frac{g}{4!}(2\pi)^d\delta^d\left(\sum_{a=1}^{4}k_a\right) \tag{2.7}$$

is the usual vertex of the commutative theory. The new vertex is not anymore invariant for the exchange of incoming propagators, but maintains invariance for cyclic permutations. As a consequence the planar and nonplanar diagrams are not necessarily equal and have to be calculated separately. In this paper we will limit ourselves to the one loop case because we are interested in the generic behaviour in the ultraviolet. Therefore we will be looking at the two diagrams described in figure 1 and the one loop corrections to the propagator. The corresponding Green's functions are

$$\begin{aligned} G^{(2)}_{\mathrm{P}} &= -\mathrm{i}\frac{g}{3}\int\frac{\mathrm{d}q^d}{(2\pi)^d}\frac{1}{(p^2-m^2)^2(q^2-m^2)} \\ G^{(2)}_{\mathrm{NP}} &= -\mathrm{i}\frac{g}{6}\int\frac{\mathrm{d}q^d}{(2\pi)^d}\frac{\mathrm{e}^{\mathrm{i}p_i\theta^{ij}q_j}}{(p^2-m^2)^2(q^2-m^2)} \end{aligned} \tag{2.8}$$

In particular we see that the planar diagram is the same as in the commutative case, thus dashing the hope that this particular noncommutative theory, with its inherent nonlocality, could solve the infinities of field theory. The persistence of some divergences is more general than the present calculation and was noted in [18] in the general framework of Connes'

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noncommutative geometry [24], while in [23] it is shown that not all divergences can be eliminated in the presence of the commutation relation (2.4).

Let us concentrate on the nonplanar diagram. For this diagram there are no ultraviolet divergences, and it is this diagram that shows the ultraviolet/infrared mixing. For high momentum p the phase in the numerator oscillates rapidly and renders the diagram convergent. However the numerator vanishes for $p \to 0$ and we have

$$\lim_{p_i \theta^{ij} \to 0} G^{(2)}_{\mathrm{NP}} = \frac{1}{2} G^{(2)}_{\mathrm{P}} \tag{2.9}$$

In [25] following the procedure set in [26] we have shown that the ultraviolet/infrared mixing persists in an unchanged way also for a variant of the Grönewold-Moyal product, the Wick-Voros product. This is naturally defined in two dimensions but can be generalized to higher dimensions. Define:

$$z_\pm = \frac{x^1 \pm \mathrm{i} x^2}{\sqrt{2}} \tag{2.10}$$

We will also use the notation

$$k_\pm = \frac{k_1 \pm \mathrm{i} k_2}{\sqrt{2}} \tag{2.11}$$

for a generic vector $\vec{k}$.

The series form of the Wick-Voros product, analog of (2.2) is

$$f \star_V g = \sum_n \left(\frac{\theta^n}{n!}\right) \partial_+^n f \partial_-^n g = f \mathrm{e}^{\theta \overleftarrow{\partial_+} \overrightarrow{\partial_-}} g \tag{2.12}$$

where

$$\partial_\pm = \frac{\partial}{\partial z_\pm} = \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial x^1} \mp \mathrm{i} \frac{\partial}{\partial x^2} \right) \tag{2.13}$$

The integral expression analog of (2.3) is

$$(f \star_V g)(x) = \frac{1}{(2\pi)^{\frac{d}{2}}} \int \mathrm{d}^d p \, \mathrm{d}^d q \tilde{f}(q) \tilde{g}(p-q) \mathrm{e}^{\mathrm{i} p \cdot x} \mathrm{e}^{-\theta q_- (p_+ - q_+)} \tag{2.14}$$

It results

$$\begin{aligned} z_+ \star_V z_- &= z_+ z_- + \theta \\ z_- \star_V z_+ &= z_+ z_- \end{aligned} \tag{2.15}$$

and therefore

$$[z_+, z_-]_{\star_V} = \theta \tag{2.16}$$

Going back to the x's, it is possible to see that this relation gives rise again to the standard commutator among the x's:

$$x^1 \star_V x^2 - x^2 \star_V x^1 = \mathrm{i}\theta \tag{2.17}$$

With the $z_\pm$ coordinates the Laplacian and the d'Alembertian are respectively $\nabla^2 = 2\partial_+ \partial_-$ and $\Box = \partial_0^2 - \nabla^2$. The integral on the plane is still a trace, but the strong condition of (2.5) is not valid anymore:

$$\int \mathrm{d}^2 z f \star_V g = \int \mathrm{d}^2 z g \star_V f \neq \int \mathrm{d}^2 z f g \tag{2.18}$$

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where by $\mathrm{d}^2 z$ we mean the usual measure on the plane $\mathrm{d}z_+\mathrm{d}z_-$. This means that the free propagator is not the same anymore as it receives a correction[1] by a factor $\mathrm{e}^{-\frac{\theta}{2}|\vec{p}|^2}$. The vertex has been calculated [25] and is

$$V_{\star_V} = V \prod_{a<b} \mathrm{e}^{-\theta k_{a-} k_{b+}} = V \prod_{a<b} \mathrm{e}^{-\frac{\theta}{2}(\vec{k_a}\cdot\vec{k_b} + i\varepsilon^{ij} k_{a_i} k_{b_j})} \tag{2.19}$$

It is then possible to calculate the one loop correction to the propagator. For the planar case we obtain

$$\begin{aligned} G^{(2)}_{\mathrm{P}} &= -\mathrm{i}\frac{g}{3}\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\mathrm{e}^{-\theta(2p_-p_+ + q_-q_+)}\mathrm{e}^{-\theta(p_-q_+ - p_-q_+ - p_-p_+ - q_-q_+ - q_-p_+ + q_-p_+)}}{(p^2-m^2)^2(q^2-m^2)} \\ &= -\mathrm{i}\frac{g}{3}\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\mathrm{e}^{-\theta p_-p_+}}{(p^2-m^2)^2(q^2-m^2)} \end{aligned} \tag{2.20}$$

In this case all the contribution due to q cancel, so that there is no change in the convergence of the integral. This is the same as in the Grönewold-Moyal case. The only difference with is again the correction to the propagator. The expression for the nonplanar case is:

$$\begin{aligned} G^{(2)}_{NP} &= -\mathrm{i}\frac{g}{6}\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\mathrm{e}^{-\theta(2p_-p_+ + q_-q_+)}\mathrm{e}^{-\theta(p_-q_+ - p_-p_+ - p_-q_+ - q_-p_+ - q_-q_+ + p_-q_+)}}{(p^2-m^2)^2(q^2-m^2)} \\ &= -\mathrm{i}\frac{g}{6}\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\mathrm{e}^{-\theta(p_-p_+ + \mathrm{i}\vec{p}\wedge\vec{q})}}{(p^2-m^2)^2(q^2-m^2)} \end{aligned} \tag{2.21}$$

This time the q contribution does not cancel completely, and there remains the exponential of the factor

$$p_-q_+ - q_-p_+ = \mathrm{i}\vec{p}\wedge\vec{q} \tag{2.22}$$

which is the same as in the Moyal case. We see that the ultraviolet behaviour of the two products is the same. The presence of the term $\mathrm{e}^{\theta p^2}$ is due to the fact that the free propagator is different in this case from the commutative theory, which in turn is a consequence of the fact that for the Wick-Voros product the property (2.5) does not hold, but the integral is still tracial ($\int \mathrm{d}x^d f \star g = \int \mathrm{d}x^d g \star f$). Apart from this difference the structure is the same in the two theories, namely the one loop diagram does not give extra contributions in the planar case, while it does in the non planar one. We will see below that this behaviour is generic for all translation invariant products.

3 General translation invariant products

In this section we first introduce a generic star product in the differential series form, and in the integral form. General star products were introduced in [27, 28] in the framework of quantization of Poisson manifolds. For our purposes a generic star product is an associative product between functions on $\mathbb{R}^d$ which depends on one or more parameters. In the limit in which these parameters vanish the product becomes the usual poinwise product. Notice

[1]In the following, for this subsection, we will be in 2+1 dimensions to ease the comparison with the Moyal product. The results are more general and dimension independent.

that we contemplate the possibility that the star product be commutative, although in general it will not be so.

We will consider two ways of expressing these generic products. In most cases (as in the Grönewold-Moyal case) these two ways coincide on a dense domain on some space of functions. The problem with expressions like (3.32) is that they are defined only at the level of formal series, and there is no certainty that one can actually find a representation of the deformed algebra of (a class of) the functions on spacetime with the noncommutative product they define. We prefer to adhere to the principle: *no deformation without representation* [29] and will present first the integral form of the product, which is more suited for our purposes. Later we will present the generic differential form as well, and will comment throughout the paper on both forms of the product.

The generalization of the star product (2.3) (or the Wick-Voros product (2.14)) is the following

$$f \star g = \frac{1}{(2\pi)^{\frac{d}{2}}} \int \mathrm{d}p^d \mathrm{d}q^d \mathrm{d}k^d \mathrm{e}^{\mathrm{i}p\cdot x} \tilde{f}(q) \tilde{g}(k) K(p,q,k) \tag{3.1}$$

Where K can be a distribution and $\tilde{f}(q)$ is the Fourier tranform of f. The product of d-vectors is understood with the Minkowski or Euclidean metric: $p \cdot x = p_i x^i$. The usual pointwise product is also of this kind for $K(p,q,k) = \delta^d(k-p+q)$. The biggest restrictions on K come from the associativity requirement which reads

$$\int \mathrm{d}k^d K(p,k,q) K(k,r,s) = \int \mathrm{d}k^d K(p,r,k) K(k,s,q) \tag{3.2}$$

This is nothing but the usual cocycle condition in the Hochschild cohomology, where the two cocycle $c \in C^2(\mathcal{A})$ is the map

$$\begin{aligned} c : (f,g) \, &\in \mathcal{A} \otimes \mathcal{A} \longrightarrow \mathcal{A} \\ c(f,g) \, &= f \star g \end{aligned} \tag{3.3}$$

$\mathcal{A}$ is the noncommutative algebra of functions with the star-product (3.1) and the coboundary operator

$$\partial : C^k(\mathcal{A}) \longrightarrow C^{k+1}(\mathcal{A}) \tag{3.4}$$

$$\begin{aligned} \partial c(f_0,\ldots,f_k) \, &= f_0 \star c(f_1,\ldots,f_k) + \sum_{i=0}^{k-1} (-1)^{i+1} c(f_0,\ldots,f_i \star f_{i+1},\ldots,f_k) \\ &+ (-1)^{k+1} c((f_0,\ldots,f_{k-1}) \star f_k . \end{aligned} \tag{3.5}$$

In order for the two-cochain (3.3) to be a two-cocycle this becomes

$$\begin{aligned} 0 = \partial c(f,g,h) \, &= f \star c(g,h) - c(f\star g,h) + c(f,g\star h) - c(f,g) \star h \\ &= 2\left(f \star (g \star h) - (f \star g) \star h\right) \end{aligned} \tag{3.6}$$

that is (3.2).

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We now proceed to the discussion on translation invariant products. Defining the translation by a vector a by $\mathcal{T}_a(f)(x) = f(x+a)$, by translation invariant product we mean the property

$$\mathcal{T}_a(f) \star \mathcal{T}_a(g) = \mathcal{T}_a(f \star g) \tag{3.7}$$

At the level of Fourier transform we have

$$\widetilde{\mathcal{T}_a f}(q) = \mathrm{e}^{\mathrm{i}ap} \tilde{f}(q) \tag{3.8}$$

For the invariance of the product (3.1) we must have

$$\begin{aligned} \mathrm{e}^{\mathrm{i}a\cdot p} \int \mathrm{d}p^d \mathrm{d}q^d \mathrm{d}k^d \mathrm{e}^{\mathrm{i}p\cdot x} \tilde{f}(q)\tilde{g}(k)K(p,q,k) = \\ = \int \mathrm{d}q^d \mathrm{d}k^d \mathrm{e}^{\mathrm{i}a\cdot q} \mathrm{e}^{\mathrm{i}a\cdot k} \mathrm{e}^{\mathrm{i}p\cdot x} \tilde{f}(q)\tilde{g}(k)K(p,q,k) \end{aligned} \tag{3.9}$$

which means that at the distributional level

$$K(p,q,k) = \mathrm{e}^{\mathrm{i}(k-p+q)\cdot a} K(p,q,k) \tag{3.10}$$

which is solved by

$$K(p,q,k) = \mathrm{e}^{\alpha(p,q)} \delta(k-p+q) \tag{3.11}$$

where α is a generic function. We will therefore consider products that can be expressed as

$$f \star g = \frac{1}{(2\pi)^{\frac{d}{2}}} \int \mathrm{d}p^d \mathrm{d}q^d \mathrm{e}^{\mathrm{i}p\cdot x} \tilde{f}(q)\tilde{g}(p-q) \mathrm{e}^{\alpha(p,q)} \tag{3.12}$$

The usual pointwise product is given by $\alpha = 0$, the Grönewold-Moyal product by $\alpha_M(p,q) = -\mathrm{i}/2\theta^{ij} q_i p_j$ and the Wick-Voros by $\alpha_V(p,q) = -\theta q_-(p_+ - q_+)$.

Associativity and the requirement that the integral is a trace impose severe constraints on the form of α. In particular from (3.2) and (3.11) the cocycle condition becomes

$$\alpha(p,q) + \alpha(q,r) = \alpha(p,r) + \alpha(p-r,q-r) \tag{3.13}$$

from this cocycle relation follow some other useful relations:

$$\begin{aligned} \alpha(p,p) &= \alpha(0,0) = \alpha(p,0) \\ \alpha(0,p) &= \alpha(0,-p) \\ \alpha(p,q) &= -\alpha(q,p) + \alpha(0,q-p) \\ \alpha(p+q,p) &= -\alpha(0,p+q) + \alpha(0,p) + \alpha(0,q) - \alpha(-p-q,-q) \end{aligned} \tag{3.14}$$

This last relation ensures also the trace property.

$$\begin{aligned} \int \mathrm{d}x^d f \star g &= \int \mathrm{d}x^d \mathrm{d}p^d \mathrm{d}q^d \mathrm{e}^{\alpha(p,q)} \mathrm{e}^{\mathrm{i}p\cdot x} \tilde{f}(q)\tilde{g}(p-q) \\ &= \int \mathrm{d}q^d \mathrm{e}^{\alpha(0,q)} \tilde{f}(q)\tilde{g}(-q) \end{aligned} \tag{3.15}$$

Another relation which will be useful in the following is

$$\alpha(p,q) = -\alpha(0,p) + \alpha(0,q) + \alpha(0,p-q) - \alpha(-p,q-p). \tag{3.16}$$

We also require the algebra to be a $*$-algebra. That is that there is a $*$ conjugation such that $f^{**} = f$ and $(f \star g)^* = g^* \star f^*$. This latter relation imposes

$$\alpha(p,q)^* = \alpha(-p,q-p) \tag{3.17}$$

Note that we do not require necessarily $f \star 1 = 1 \star f = f$, that is that the identity of the algebra is the constant function. This condition would impose

$$\alpha(p,p) = 0 \quad \text{and} \quad \alpha(p,0) = 0 \tag{3.18}$$

The $\star$ products that we are considering are in general noncommutative, but a product of the form (3.12) can be commutative. In this case we have that the restriction on the kernel K reads

$$K(p,k,q) = K(p,q,k) \tag{3.19}$$

that is the cocycle c is a coboundary

$$c(f,g) = \partial b(f,g) = f \star b(g) + g \star b(f) - b(f \star g) \tag{3.20}$$

with the cochain b given by the identity map. In terms of α the coboundary condition becomes

$$\alpha(p,q) = \alpha(p,p-q) \tag{3.21}$$

3.1 Cohomology

It is possible to define an "α-cohomology" with respect to which α is a 2-cocycle, while it becomes a coboundary for a commutative product. $\alpha \in A^2(\tilde{\mathcal{A}})$ is the map

$$\alpha : (p,q) \in \tilde{\mathcal{A}} \otimes \tilde{\mathcal{A}} \longrightarrow \tilde{\mathcal{A}} \tag{3.22}$$

with $\tilde{\mathcal{A}}$ the algebra of Fourier transforms (to be precise α is defined on translations, realised as linear functions in $\tilde{\mathcal{A}}$) and the coboundary operator

$$\partial : A^k(\tilde{\mathcal{A}}) \longrightarrow A^{k+1}(\tilde{\mathcal{A}}) \tag{3.23}$$

$$\partial\gamma(p_0,\ldots,p_k) = \sum_{i=0}^{k} (-1)^i \gamma(p_0,\ldots,p_{\hat{i}},p_{i+1},\ldots,p_k) - (-1)^k \gamma(p_0-p_k, p_i-p_k,\ldots,p_{k-1}-p_k) \tag{3.24}$$

In order for α in (3.22) to be a two-cocycle this becomes

$$0 = \partial\alpha(p,q,r) = \alpha(q,r) - \alpha(p,r) + \alpha(p,q) - \alpha(p-r,q-r) \tag{3.25}$$

that is (3.13). A straightforward calculation verifies that $\partial^2 = 0$. Thus, the associativity condition (3.13) is a 2-cocylce condition in the α cohomology. Analogously the commutativity condition can be shown to be a coboundary condition. Indeed, for α to be a coboundary it has to be

$$\alpha(p,q) = \partial\beta(p,q) = \beta(q) - \beta(p) + \beta(p-q) \tag{3.26}$$

which implies the commutativity condition (3.21), that is $\alpha(p,q) = \alpha(p, p-q)$.

The Grönewold-Moyal and Wick-Voros products, both noncommutative, are respectively given by,

$$\alpha_M(p,q) = -\frac{\mathrm{i}}{2}\theta^{ij} q_\mathrm{i}(p_j - q_j) = \frac{i}{2}\theta p \wedge q \tag{3.27}$$

and

$$\alpha_V(p,q) = -\theta q_-(p_+ - q_+) = \alpha_M(p,q) - \frac{\theta}{2}(p-q)\cdot q \tag{3.28}$$

which are both cocyles in the α-cohomology and, more interestingly, differ by a term which is a α-coboundary, according to (3.26) with β so defined

$$\beta(p) = p^2 \tag{3.29}$$

Indeed we easily verify that

$$\alpha_V(p,q) = \alpha_M(p,q) + \frac{\theta}{4}\partial\beta(p,q) \tag{3.30}$$

With a symbolic manipulation programme and a little work is not difficult to construct viable polynomial α's. For example the following expression in two dimensions gives rise to an associative product:

$$\begin{aligned}\alpha &= \gamma_1 p_2 q_1 + \gamma_2 p_1 q_2 - (\gamma_1+\gamma_2) q_1 q_2 + \beta_1 (p_2 q_2^2 - p_2^2 q_2) \\ &\quad + \beta_2 \left(\frac{p_2^2 q_1 - p_1 q_2^2}{2} + p_1 p_2 q_2 - p_2 q_1 q_2 \right)\end{aligned} \tag{3.31}$$

for arbitrary $\gamma_1, \gamma_2, \beta_1$ and β_2.

3.2 The differential form of the product

The second form is a generalization of the expression (2.2) and it is a series which depends on a "small" parameter which we call again θ:

$$f \star g = \sum_{r=0}^{\infty} C_r(f,g)\theta^r \,, \tag{3.32}$$

To recover the original commutative product in the limit $\theta \to 0$ we need to impose that $C_0(f,g) = fg$. To ensure associativity the remaining C_r's have to satisfy the following properties,

$$\begin{aligned} &f C_r(g,h) - C_r(fg,h) + C_r(f,gh) - C_r(f,g)h \\ &\qquad = \sum_{j+k=r} \left(C_j(C_k(f,g),h) - C_j(f, C_k(g,h)) \right) \,, \end{aligned} \tag{3.33}$$

for all $j, k, r > 0$. The generalization to the multiparameter case is easily done considering θ and the C's to have indices which are summed over.

A possible problem with this form of products is that it is defined on the space of formal series in the coordinates, and there is in general no control on the convergence of the series after the product has been taken. Moreover not always the differential form is useful for field theory. The quantity $C_1(f,g) - C_1(g,f)$ gives a Poisson structure on the space which is important for quantization. One defines the Poisson structure as:

$$\{f, g\} = C_1(f,g) - C_1(g,f) = \Lambda ij \partial_i f \partial_j g \tag{3.34}$$

where

$$\Lambda^{ij} = \frac{1}{2}\left(C_1(x^i, x^j) - C_1(x^j, x^i)\right) \tag{3.35}$$

Notice that if $C_1(f,g) = C_1(g,f)$ then the product is commutative. The proof is the following. First consider (3.33) for $r = 2$ and $h = f = x^n$ and $g = x^m$. Then relation (3.33) becomes

$$\begin{aligned} & fC_2(g,f) - C_2(fg,f) + C_2(f,gf) - C_2(f,g)f \\ & \quad = x^n(C_2(x^m,x^n) - C_2(x^n,x^m)) - C_2(x^{n+m},x^n) + C_2(x^n,x^{n+m}) \\ & \quad = (C_1(C_1(f,g),f) - C_1(f,C_1(g,f))) = 0 \end{aligned} \tag{3.36}$$

because of the symmetry of C_1. The second line of the above equation, has to hold for all x's and therefore it must be $C_2(x^{n+m}, x^n) = C_2(x^n, x^{n+m})$ for generic n, m. This implies that $C_2(f,g) = C_2(g,f)$. It is then possible to prove exactly in the same way that if $C_l(f,g) = C_l(g,f)$ for $l < r$ then all the terms in the r.h.s. of (3.33) pairwise cancel, and we are left to the equivalent of (3.36) with a generic r, and then analogously prove that $C_r(f,g) = C_r(g,f)$.

Since the c_i's are differential operators the product is translationally invariant if and only if the C's are combinations of derivatives only. In this case

$$\Lambda^{ij} = \frac{1}{2}[x^i, x^j]_\star \tag{3.37}$$

There is a notion of equivalence which says that two star products are to be considered as equivalent if there exists a map T such that

$$T(f) \star T(g) = T(f \star' g). \tag{3.38}$$

According to this the Grönewold-Moyal and Wick-Voros products are equivalent, the map T being given by $T = \mathrm{e}^{-\frac{\theta}{4}\nabla^2}$. A general result of Kontsevich (in the context of formal series) [12] proves that two products with the same Poisson structure are equivalent. We have seen an instance of such an equivalence while calculating the UV behaviour of Feynman diagrams at one-loop. We have seen that, although the Green's functions are different for Moyal and Voros products, the UV behaviour is the same as well as the UV/IR mixing. We will see in the next section that this is a generic feature of translation invariant products and what counts is the cohomology class of α in the α-cohomology.

4 UV/IR mixing for general products

We are now ready to calculate the two point functions at one loop. In this paper we are only interested to the ultraviolet properties of the generalized products. The presence of the deformed product also changes the propagator and it may alter the S-matrix. A full analysis of a scattering process however requires to take into account issues of symmetry, and the proper definition of the incoming states. We have considered [25] the issue for the case in which the product is coming from a twisted symmetry [19–21] (for a review see [22]) and found that a proper treatment of the incoming states and of the symmetries implies that there is no difference between the Grönewold-Moyal and Wick-Voros products.

We now proceed to the calculation of the loop contribution. We first have to give the free propagator, which is

$$\tilde{G}_0^2(p) = \frac{e^{-\alpha(0,p)}}{p^2 - m^2} \,. \tag{4.1}$$

The presence of the exponential in the propagator alters its properties. The analysis of this (free) propagator and its role in the S-matrix involves a proper definition of the asymptotic states and their normalization. Since in this paper we are only interested in the corrections of the propagator due to the loop expansion, and the ultraviolet/infrared mixing, we will not discuss this issue. We just comment that in [25] it is shown that in the case of the Wick-Voros product, in the context of twisted deformations, the exponential is absorbed in the normalization of the in and out states.

In order to calculate the vertex, let us write down the interacting term of the action in momentum space. Using the definition of the product and the fact that the integral is a trace we have

$$\begin{aligned} S_{\text{int}} &= \frac{g}{4!}\int \mathrm{d}x^d\, \mathrm{d}k_1^d\, \mathrm{d}k_2^d\, \mathrm{d}k_3^d\, \mathrm{d}k_4^d\, \tilde{\phi}(k_2)\tilde{\phi}(k_1-k_2)\tilde{\phi}(k_4)\tilde{\phi}(k_3-k_4) \\ &\qquad e^{\alpha(k_1,k_2)}e^{\alpha(k_3,k_4)}e^{ik_1\cdot x}\star e^{ik_3\cdot x} \\ &= \frac{g}{4!}\int \mathrm{d}k_1^d\, \mathrm{d}k_2^d\, \mathrm{d}k_3^d\, \mathrm{d}k_4^d\, \tilde{\phi}(k_2)\tilde{\phi}(k_1-k_2)\tilde{\phi}(k_4)\tilde{\phi}(k_3-k_4) \\ &\qquad e^{\alpha(k_1,k_2)}e^{\alpha(k_3,k_4)}\int \mathrm{d}k^d\, e^{\alpha(0,k)}\delta(k_1-k)\delta(k_3+k). \end{aligned} \tag{4.2}$$

So

$$\begin{aligned} S_{\text{int}} &= \frac{g}{4!}\int \mathrm{d}k_1^d\, \mathrm{d}k_2^d\, \mathrm{d}k_3^d\, \mathrm{d}k_4^d\, \tilde{\phi}(k_2)\tilde{\phi}(k_1-k_2)\tilde{\phi}(k_4)\tilde{\phi}(k_3-k_4) \\ &\qquad e^{\alpha(k_1,k_2)}e^{\alpha(k_3,k_4)}e^{\alpha(0,k_1)}\delta(k_1+k_3) \\ &= \frac{g}{4!}\int \mathrm{d}k_1^d\, \mathrm{d}k_2^d\, \mathrm{d}k_3^d\, \mathrm{d}k_4^d\, \tilde{\phi}(k_2)\tilde{\phi}(k_1-k_2)\tilde{\phi}(k_4)\tilde{\phi}(k_3-k_4) \\ &\qquad e^{\alpha(k_1,k_2)+\alpha(k_3,k_4)+\alpha(0,k_1)}\delta(k_1+k_3) \\ &= \frac{g}{4!}\int \mathrm{d}k_1^d\, \mathrm{d}k_2^d\, \mathrm{d}k_3^d\, \mathrm{d}k_4^d\, \tilde{\phi}(k_1)\tilde{\phi}(k_2)\tilde{\phi}(k_3)\tilde{\phi}(k_4) \\ &\qquad e^{\alpha(k_1+k_2,k_1)+\alpha(k_3+k_4,k_3)+\alpha(0,k_1+k_2)}\delta(k_1+k_2+k_3+k_4). \end{aligned} \tag{4.3}$$

Therefore the vertex is given by

$$V_\star = V_0 e^{\alpha(k_1+k_2,k_1)+\alpha(k_3+k_4,k_3)+\alpha(0,k_1+k_2)}, \tag{4.4}$$

where V_0 is the ordinary vertex defined in (2.7). We now proceed to the calculation of the four-point Green's function to the tree level. To this end, we must attach to the vertex four propagators. So we have up to a constant

$$\tilde{G}^{(4)} = e^{\alpha(k_1+k_2,k_1)+\alpha(k_3+k_4,k_3)+\alpha(0,k_1+k_2)} \prod_{a=1}^{4} \frac{e^{-\alpha(0,k_a)}}{k_a^2 - m^2} \delta\left(\sum_{a=1}^{4} k_a\right) \tag{4.5}$$

$$= \frac{e^{\alpha(k_1+k_2,k_1)+\alpha(k_3+k_4,k_3)+\alpha(0,k_1+k_2)-\sum_{a=1}^{4}\alpha(0,k_a)}}{\prod_{a=1}^{4}(k_a^2 - m^2)} \delta\left(\sum_{a=1}^{4} k_a\right). \tag{4.6}$$

Consider now the two diagrams of figure 1. For the planar case (a) the correction is obtained using three propagators (4.1), one with momentum p, one with momentum $-p$, one with momentum q and the vertex (4.4) with assignments $k_1 = -k_4 = p$ and $k_2 = -k_3 = q$ and, of course, the integration in q. We have up to a constant

$$\begin{aligned} G_{\mathrm{P}}^{(2)} &= \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-\alpha(0,p)-\alpha(0,-p)-\alpha(0,q)}}{(p^2-m^2)^2(q^2-m^2)} \mathrm{e}^{\alpha(p+q,p)+\alpha(-p-q,-q)+\alpha(0,p+q)} \\ &= \int \mathrm{d}q^d \, \frac{e^{-\alpha(0,p)-\alpha(0,-p)-\alpha(0,q)+\alpha(p+q,p)+\alpha(-p-q,-q)+\alpha(0,p+q)}}{(p^2-m^2)^2(q^2-m^2)} \\ &= \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-2\alpha(0,p)-\alpha(0,q)+\alpha(p+q,p)+\alpha(-p-q,-q)+\alpha(0,p+q)}}{(p^2-m^2)^2(q^2-m^2)} \\ &= \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-\alpha(0,p)}}{(p^2-m^2)^2(q^2-m^2)} \end{aligned} \tag{4.7}$$

where we used the last of (3.14). We see that with respect to the commutative case the only correction is in the factor $\mathrm{e}^{-\alpha(0,p)}$ which is the correction of the free propagator. The ultraviolet divergences of the loop are the same and therefore the short distance physics is unaffected (in this aspect) by the star product. The correction of the free propagator can then be reabsorbed in the S-matrix as done in [25].

Consider now the non-planar case in figure 1(b). The structure is the same as in the planar case, but this time the assignments are

$$k_1 = -k_3 = p \quad \text{and} \quad k_2 = -k_4 = q. \tag{4.8}$$

We have up to a constant

$$\begin{aligned} G_{\mathrm{NP}}^{(2)} &= \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-\alpha(0,p)-\alpha(0,-p)-\alpha(0,q)}}{(p^2-m^2)^2(q^2-m^2)} \mathrm{e}^{\alpha(p+q,p)+\alpha(-p-q,-p)+\alpha(0,p+q)} \\ &= \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-\alpha(0,p)-\alpha(0,-p)-\alpha(0,q)+\alpha(p+q,p)+\alpha(-p-q,-p)+\alpha(0,p+q)}}{(p^2-m^2)^2(q^2-m^2)} \\ &= \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-2\alpha(0,p)-\alpha(0,q)+\alpha(p+q,p)+\alpha(-p-q,-p)+\alpha(0,p+q)}}{(p^2-m^2)^2(q^2-m^2)} \\ &= \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-\alpha(0,p)+\alpha(p+q,p)-\alpha(p+q,q)}}{(p^2-m^2)^2(q^2-m^2)} \end{aligned} \tag{4.9}$$

since by using again (3.16) we have

$$\begin{aligned}\alpha(-p-q,-p) &= -\alpha(0,-p-q)+\alpha(0,-p)+\alpha(0,-q)-\alpha(p+q,q) \\ &= -\alpha(0,p+q)+\alpha(0,p)+\alpha(0,q)-\alpha(p+q,q).\end{aligned} \tag{4.10}$$

The one-loop corrections to the propagator in the non-planar case can be rewritten as

$$G^{(2)}_{NP} = \int \mathrm{d}q^d \, \frac{\mathrm{e}^{-\alpha(0,p)+\omega(p,q)}}{(p^2-m^2)^2(q^2-m^2)}, \tag{4.11}$$

where we define

$$\omega(p,q) = \alpha(p+q,p) - \alpha(p+q,q). \tag{4.12}$$

For the Groönewold-Moyal product this term is the phase $\mathrm{i}p_i\theta^{ij}q_j$.

This function has some useful property. First of all, it satisfies the 2-cocycle condition (3.13). Moreover,

$$\omega(p,p) = 0 \tag{4.13}$$

$$\omega(p,0) = 0 \tag{4.14}$$

$$\omega(0,p) = 0 \tag{4.15}$$

$$\omega(p,q) = -\omega(q,p) \qquad \text{antisymmetry} \tag{4.16}$$

$$\omega(-p,-q) = \omega(p,q) \qquad \text{parity} \tag{4.17}$$

$$\omega(-p,q) = \omega(p,-q) \tag{4.18}$$

$$\omega(p,q) = \omega(p-q,p). \tag{4.19}$$

From (3.13) we have

$$\alpha(p+q,p) = \alpha(p+q,r) - \alpha(p,r) + \alpha(p+q-r,p-r) \tag{4.20}$$

and by setting $r=q$ we get

$$\omega(p,q) = \alpha(p,p+q) - \alpha(p,q). \tag{4.21}$$

This quantity vanishes if the product is commutative because of the condition (3.21), that is ω is a 2-cocycle which is not a coboundary. This means that the nonplanar diagram captures the noncommutativity of the product, or, it only depends on the α-cohomology class. In other words no change in the ultraviolet can come from a commutative product (an α-coboundary).

We now prove that the contribution to the one loop diagram must necessarily be of the form $p_i\theta^{ij}q_j$ and that it depends on the Poisson structure induced by the star product. We will only need the rather mild assumption that α (and therefore ω) can be Taylor expanded in a power series of p and q. The parity relation (4.17) requires the series to be composed only of even monomials. Let us express the function ω with a multi-index notation

$$\omega(p,q) = \sum_{\vec{i}\vec{j}} a_{\vec{i}\vec{j}}\, p^{\vec{i}} q^{\vec{j}} \tag{4.22}$$

where $\vec{i} = (i_1, \dots i_d)$ and

$$p^{\vec{i}} = p_1^{i_1} p_2^{i_2} \dots p_d^{i_d} \tag{4.23}$$

If we now use relation (4.19) we have that it must be

$$\sum_{\vec{i}\vec{j}} a_{\vec{i}\vec{j}}\, q^{\vec{i}}(p^{\vec{j}} - (p-q)^{\vec{j}}) = 0 \tag{4.24}$$

this condition, because of the independence of p and q implies that the coefficient a must vanish *except* in the case in which all of the j_a's but one vanish. In this case the antisymmetry of the a's ensures that (4.24) vanishes without putting further constraints on the coefficient. Using antisymmetry the same reasoning can be done for the first multi-index and this shows that the term appearing in the one loop amplitude is of the kind $\omega(p,q) = \mathrm{i}\theta^{ij}p_i q_j$. Where we added the imaginary unit to be consistent with the usual notation. Using the relation (3.17) is possible to see that θ must be real.

In fact the expression which appears is the one related to the commutator of the coordinates. A straightforward calculation gives

$$x^i \star x^j - x^j \star x^i = -\mathrm{i}\frac{\partial\alpha}{\partial p_i}(0,0)x^j + \mathrm{i}\frac{\partial\alpha}{\partial p_j}(0,0)x^i - \frac{\partial^2\alpha}{\partial p_i\partial q_j}(0,0) + \frac{\partial^2\alpha}{\partial p_j\partial q_i}(0,0) \tag{4.25}$$

The first two terms vanish because α has no linear term (we must justify this from associativity), while the second gives the coefficients of the quadratic terms in the expansion of α antisymmetrised. On the other side we have established that ω is quadratic as well and expressing

$$\alpha(p,q) = \alpha_{ij}p^i q^j + \dots \tag{4.26}$$

where $\dots$ are terms cubic and above, we have

$$\omega(p,q) = \mathrm{i}\theta^{ij}p_i q_j = \alpha^{ij}(p_i + q_i)p_j - \alpha^{ij}(p_i+q_i)q_j = \alpha^{ij}(p_i+q_i)(p_j - q_j) \tag{4.27}$$

imposing the condition (3.18) makes only the mixed terms survive on the r.h.s., and the quadratic mixed terms are the ones which appear in (4.25).

We have shown that the term appearing in the exponent of the nonplanar diagram (4.9) is just the commutator of the x's multiplied by the external and internal momenta. The Grönewold-Moyal and Wick-Voros cases are therefore generic, their behaviour is replicated by all translationally invariant products.

Therefore we have shown that products with the same Poisson structure (and hence the same commutator) which are equivalent in the sense of Konsevitch, have the same structure of infrared/ultraviolet mixing. We have also noted that equivalence in the sense of (3.38) does not a priori mean physical equivalence. The propagators and Green's functions are in general different, as for the Grönewold-Moyal and Wick-Voros products, where the Green's functions are not the same. In this case however, using the fact that both products come from a Drinfeld twist [30], and therefore have a deformed symmetry by a quantum group [19, 21, 22], it can be shown [25] that the S-matrix is the same.

In fact it is easy to see that the general translational invariant product (3.12), in the case of α analytic comes from a Drinfeld twist. Expressing

$$\alpha(p,q) = \sum \vec{i},\vec{j}\alpha_{\vec{i},\vec{j}}q^{\vec{i}}(p-q)^{\vec{j}} \tag{4.28}$$

the product comes from the Drinfeld twist

$$\mathcal{F} = \exp\left(-\sum_{\vec{i},\vec{j}} \alpha_{\vec{i},\vec{j}}\partial_x^{\vec{i}} \otimes \partial_y^{\vec{j}}\right) \tag{4.29}$$

then the product is

$$\begin{aligned} f \star g &= \mathrm{e}^{\sum_{\vec{i},\vec{j}} \alpha_{\vec{i},\vec{j}}\partial_x^{\vec{i}}\partial_y^{\vec{j}} f(x)g(y)}\Big|_{x=y} \\ &= \int \mathrm{d}p\mathrm{d}q \mathrm{e}^{\sum_{\vec{i},\vec{j}} \alpha_{\vec{i},\vec{j}} p^{\vec{i}} q^{\vec{j}}} \tilde{f}(p)\tilde{g}(q)\mathrm{e}^{\mathrm{i}(x(p+q))} \end{aligned} \tag{4.30}$$

and the usual expression (3.12) is obtained with a change of variables.

5 Conclusions

In this paper we have shown, with an explicit calculation, that the Ultraviolet/Infrared mixing found for the Grönewold-Moyal and Wick-Voros products is a generic feature of translationally invariant associative star products. The vertex is changed by an exponential which maintains invariance for cyclic permutation of the external momenta but not for arbitrary exchanges. Therefore the planar and nonplanar diagrams behave differently.

The planar diagrams have corrections to the propagator which are unchanged with respect to the usual case. The nonplanar diagrams on the other side present the phenomenon of Ultraviolet/Infrared mixing. For high internal momentum the ultraviolet divergences are damped by a phase, but these divergences reappear in the infrared (low incoming momentum). The phase appearing in the exponent in the nonplanar diagram is the one related to the commutator of the coordinates. In a sense it may be said that the mixing is given (in this translationally invariant case) by the Poisson structure of the underlying space. This is non trivial, because the Green's functions and the propagators of the theory are in general different. What remains the same is the short distance behaviour. Our calculation confirms the heuristic argument that the mixing is a manifestation of the spacetime version of Heisenberg's uncertainty, given by the Poisson structure.

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